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Turning Lamb Mode based Crack Growth Prediction for G_{IC} Determination in Laminated Composites

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Abstract

A turning Lamb mode propagates from one sub-laminate to the other at a delaminated region. In the present work, an attempt has been made to explore the use of the fundamental symmetric and anti-symmetric Turning Lamb modes for the prediction of crack growth for the determination of the strain energy release rate (G_{IC}) in a laminated composite specimen. Expressions based on Time-of-Flight were derived to estimate the crack growth. The proposed methodology has been validated through numerical simulations.

1. Introduction

The extensive use of fibre-reinforced polymer composites as structural engineering materials in automotive, aerospace and military applications is due to their excellent mechanical properties, impact resistance, light weight and their amenability to be tailored as per external loads [Autar Kaw, 1997]. Since laminated composites have weak out-of-plane strength properties, they are prone to delamination type damages [Sridharan, 2010]. Delamination, being debilitating damage, reduces the life of the component and its structural integrity. Typically, delamination acts as a crack-like defect between the laminae interface which grows when the structure is subjected to tensile loads. The fracture toughness of the polymer matrix and the interfacial shear strength between the matrix and the fibre are of prime importance in monitoring the through-thickness properties of the composites.

Lamb waves, also known as guided waves, propagate in the sagittal plane in a traction-free plate. For isotropic media, Helmholtz decomposition is used to obtain individual equations for longitudinal and shear waves [Rose, 1999] from three coupled partial differential equations, which are stress equilibrium equations expressed in terms of displacements. Lamb waves can propagate long distances in plate-like and cylindrical structures. They also have through-thickness displacements/ stresses, and provide information along the line-of-sight. Lamb waves are dispersive; viz. velocity depends on the product of frequency and thickness. Depending on the relationship between the displacement profiles and thickness, the modes are classified into symmetric (S_n) and anti-symmetric (A_n) modes. The specific order (represented by the subscript 'n') of these two modes depends on the excitation frequency and thickness of an isotropic plate. However, in an anisotropic plate, it also depends on the direction of propagation. When a Lamb wave mode encounters a defect in its path of propagation, the wave reflects, scatters and the mode converts into other mode types. Hence, these waves can be used for Non-destructive Evaluation (NDE) [Islam *et al*, 1994] as well as Structural

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Health Monitoring (SHM) [Raghavan, 2007] of laminated composite structures, particularly using attached or embedded wafer piezo sensors. When a Lamb mode propagates through a defective region, one may find other modes, in addition to the excited mode in the received signal. It is difficult to separate various modes from the received signal. Owing to this complexity of the received signal it is tedious to characterize defects using Lamb waves. By selecting the excitation frequency in a less dispersive region, where only the fundamental Lamb modes, S_o (primary symmetric) and A_o (primary anti-symmetric) exist, the complexity of the signal can be reduced, to some extent. Numerical simulations help greatly in understanding the interactions of guided waves with various defects in composite plates.

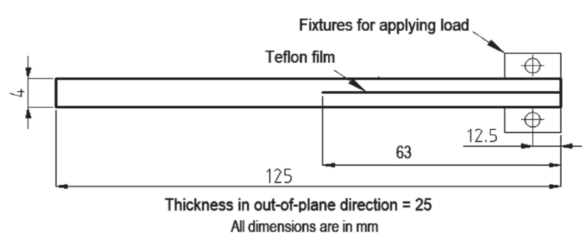

Fig. 1 Dimensions of DCB specimen used for G_{IC} test.

In addition to NDE and SHM applications some researchers explored the use of Lamb waves for the reconstruction of the elastic moduli of anisotropic plates. Rogers [1995] described a method for the measurement of the elastic moduli of isotropic plates using Rayleigh-Lamb waves produced by using a pair of variable-angle contact transducers in pitch-catch mode. From this technique, Young's modulus and Poisson's ratio were estimated. Rao [1997] reconstructed the elastic moduli of a unidirectional composite by reducing the error between theoretical and experimental Lamb wave dispersion curves. Vishnuvardhan *et al* [2007] attempted to reconstruct all nine elastic moduli of orthotropic plates using a Single-Transmitter-Multiple-Receiver (STMR) compact Structural Health Monitoring (SHM) array. Phase velocities of fundamental symmetric and anti-symmetric Lamb waves were used in an inversion technique based on genetic algorithms. Karim *et al* [1990] proposed an inversion scheme to invert Leaky Lamb wave (LLW) velocity data using a simplex algorithm to approximate the elastic moduli and thickness of an adhesive layer between two aluminum plates and the elastic moduli of a unidirectional GFRP composite laminate.

As per ASTM D5528, the strain energy release rate (G_{IC}) of a laminated composite is estimated using a Double Cantilever Beam (DCB) specimen having a semi-infinite delamination/crack as shown in Figure 1. When a load is applied at the ends, the crack starts propagating and the load is measured for every one milli meter (mm) of crack growth till the crack reaches 68 mm size. Later, the load required for every five mm of crack growth is measured. A magnifying lens is used to measure the crack growth. In the present work, a technique based on Time-of-Flight (ToF) of transmitted and Turning Lamb Modes (TLM) is proposed for the measurement of crack growth.

2. Estimation of Crack Growth using Turning Lamb Modes

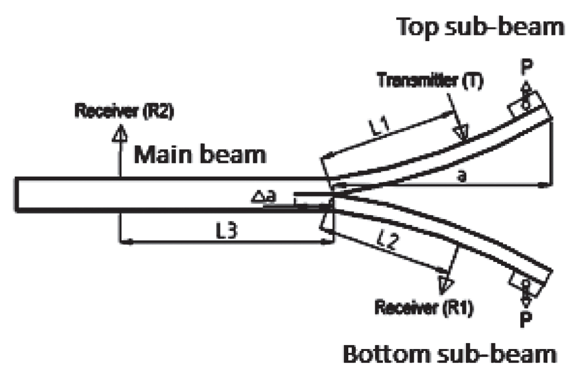

Fig. 2 Locations of the transmitter and receivers.

Figure 2 shows the locations of the transmitter and receivers, used for the transmission and reception of Lamb modes, on a DCB specimen. Transmitter 'T' was mounted over the top sub-beam and receiver R_1 was mounted over the bottom sub-beam. The initial crack size in DCB was *a*. The distances between 'T' and the crack tip, and R_1 and the crack tip, are L_1 and L_2 respectively. The Lamb mode (either A_o or S_o) is generated by the transmitter over the top sub-beam incidents at the crack tip. This mode takes a 'U' turn and starts propagating from the top sub-laminate to the bottom sub-laminate. This is the Turning Lamb Mode [Ramadas *et al*, 2011] Receiver 'R_1' captures the

TLM. Since the lay-up and thickness in cantilever beams (sub-beams) is the same, the TLM propagates with the same velocity in both the sub-beams. The distance travelled by the TLM to reach R_1 from T is $L_1 + L_2$. Say, the arrival time of this mode at receiver R_1 is t_o^1. The incident Lamb mode at the crack tip transmits into the main beam as well. When a load is applied, the crack size becomes $a + \Delta a$. In this case, the Lamb mode travels $L_1 + L_2 + 2\Delta a$ distance to reach the receiver, R_1. The arrival times of the Lamb modes at receivers R_1 and R_2 are t_d^1 and t_d^2 respectively.

The difference between the arrival times of Lamb modes at R_1 when crack sizes are $a + \Delta a$ and a is given by,

$$t_\delta^1 - t_o^1 = \Delta t^1 = \frac{L_1 + L_2 + 2\Delta a}{V_{gs}} - \frac{L_1 + L_2}{V_{gs}} \qquad (1)$$

$$\Delta t^1 = \frac{2\Delta a}{V_{gs}} \qquad (2)$$

where, V_{gs} is the group velocity of the Lamb mode in the sub-beam. Since the frequency of excitation of the Lamb modes, thickness of the sub-beams and material properties of the UD laminate are known, the group velocities of the Lamb modes can be computed using a dispersion relation or they can be obtained from DISPERSE [Lowe, 2003].

The difference in the arrival times of the Lamb modes when the crack grows by an incremental size of Δa can be calculated by capturing the signals when the size of the cracks are a and $a + \Delta a$. Once the group velocities and the difference in the arrival times of the Lamb modes are known, crack growth can be estimated using equation (2).

3. Numerical Modeling

Numerical simulations were carried out using the Finite Element Method (FEM). The specifications of the model used for numerical modeling were as follows – length 125 mm, thickness of the main laminate 3.96 mm, thickness of the sub-laminate 1.98 mm and initial crack size 63 mm. These dimensions were chosen as per ASTM D5528. The material used was glass/epoxy, whose properties are tabulated in Table 1. The element used was an eight-node (quadratic) 2D plane element with two translational Degrees of Freedom (DoF) at each node. This element belongs to the Serendipity family. Attenuation was not considered in numerical modeling. The selection of the time step size and the element size were selected as per the reference (Yang *et al*, 2006). The size of element was 0.165 mm in thickness direction and 0.25 mm in length direction. The size of the time step was 10 nano sec. Time marching was carried out using Newmark's time integration technique.

Table 1 – Properties of GFRP lamina

Material	E_{11} (GPa)	E_{22} (GPa)	υ_{13}	υ_{23}	G_{13} (GPa)	ρ kg/m³
Glass/Epoxy	44.68	6.90	0.280	0.355	2.54	1990

The excitation frequency and number of cycles were 200 kHz and five respectively, which correspond to 25 μs of excitation duration. The excitation pulse was a tone burst modulated with a Hanning window. Two sets of numerical simulations were carried out. In the first set, the TLM used for the estimation of crack growth was the fundamental symmetric Lamb mode (S_o) and in the second set, the fundamental anti-symmetric (A_o) Lamb mode was used.

3.1 S_o Mode

Before the specimen was subjected to loading, the initial crack size was a. The S_o mode in the top sub-laminate was excited by giving in-plane displacements across the thickness. An A-scan obtained at receiver R_1 is shown in Fig 3(a). A video envelope was fitted over each Lamb mode. The peak of the video envelope was taken as the arrival time of the whole wave group. The arrival time of this wave group was taken as the reference (t_o^1) for the estimation of crack size. Now, the crack size was extended by one more mm by applying an external load. Fig 3(b) shows an A-scan obtained at receiver R_1 when the crack size was a+1 (= 64 mm) mm. Similar numerical simulations were carried out for other crack sizes as well. Table 2 lists the arrival times of the wave groups for different crack sizes and the estimated crack sizes using equation (2).

3.2 A_o Mode

An A_o mode was excited by giving out-of-

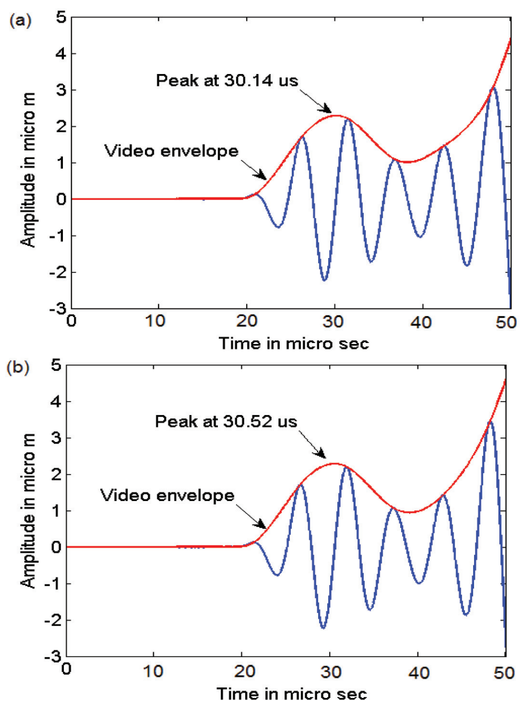

Fig. 3 A-scans fitted with video envelopes captured at R1 for crack sizes (a) 63 mm (b) 64 mm.

plane displacements across the thickness. In this case also, the arrival time of each wave group was obtained by fitting a video envelope. Table 3 lists the arrival times of the wave groups and estimated crack sizes using equation (2).

4. Results and Discussion

Numerical simulations were carried out to study the feasibility of employing the fundamental Turning Lamb Modes for the estimation of crack growth in the determination of G_{IC}. Tables 2 and 3 summarize the estimated crack growth using S_o and A_o TLMs respectively. It is noticed that the crack growth estimated by making use of the A_o Lamb mode is more accurate than the S_o mode. This is because of the following fact. Equation (2) states that for a given crack size, the difference in arrival time is inversely proportional to the group velocity. If a Lamb mode having higher group velocity is employed, it results in a smaller difference in arrival times. Since the group velocity of the A_o Lamb mode is lower than that of the S_o Lamb mode, the difference in the arrival times of the A_o mode is higher than those of the S_o Lamb mode, for a given crack growth as shown in Tables 2 and 3. In other words, a lower group velocity TLM is more sensitive than a high velocity turning Lamb mode for a given crack growth.

When a Lamb mode is incident at the crack tip, in addition to the propagation of the Turning Lamb mode, it transmits into the main beam. Receiver 'R_2' captures the transmitted Lamb mode in the main beam. The distance travelled by the transmitted Lamb mode to reach R_2 from T is $L_1 + L_3$. Say, for example, the arrival time of this mode at receiver R_2 is t_o^2. When the load was applied, the crack size became $a + \Delta a$. In this case, the Lamb mode travels a distance of $L_1 + L_3 - \Delta a$, to reach receiver R_2. The difference between the arrival times of the Lamb modes at R_2 when the crack sizes are $a + \Delta a$ and a is given by,

$$t_\delta^2 - t_o^2 = \Delta t^2 = \left(\frac{L_1 + \Delta a}{V_{gs}} + \frac{L_3 - \Delta a}{V_{gm}}\right) - \left(\frac{L_1}{V_{gs}} + \frac{L_3}{V_{gm}}\right) \tag{3}$$

$$\Delta t^2 = \Delta a\left(\frac{1}{V_{gs}} - \frac{1}{V_{gm}}\right) \tag{4}$$

Table-2 Estimation of crack growth using S_o Lamb mode. Group velocity of S_o mode in sub-laminate = 4735.8 m/s

Crack size, *a*, in mm	Increase in crack size, δ in mm	Arrival time in µs	Difference in arrival time, Δt in µs	Estimated increase in crack size using equation(2)
63	0	30.14	0	-
64	1	30.52	0.38	0.90
65	2	30.93	0.79	1.87
66	3	31.34	1.20	2.84
67	4	31.78	1.64	3.88
72	9	34.03	3.89	9.21

Table -3 Estimation of crack growth using A_o Lamb mode. Group velocity of A_o mode in sub-lamin2ate = 1133.7 m/s

Crack size, *a*, in mm	Increase in crack size, *d* in mm	Arrival time in ms	Difference in arrival time, *Dt* in ms	Estimated increase in crack size using equation(2)
63	0	91.16	0	-
64	1	92.84	1.68	0.95
65	2	94.61	3.45	1.95
66	3	96.28	5.12	2.90
67	4	98.02	6.86	3.89
72	9	106.62	15.46	8.76

Where, V_{gs} and V_{gm} are group velocities in the sub-beams and main beam respectively. The selection of excitation frequency is of immense importance when transmitted Lamb modes are used for the estimation of crack growth. For a given crack growth (Δa), if the excitation frequency is selected in such a manner that the difference in group velocities ($V_{gs} - V_{gm}$) is low, the difference in arrival times (t^2) is also low. The limitation on Δt^2 comes from the sampling frequency of the digital data. For prediction of delaminations of smaller size, the sampling frequency should be sufficiently high.

If the difference in the arrival times of the Lamb mode for crack sizes a and $a+\Delta a$ and group velocities are known, then, either equation (2) or (4) can be used to predict the increase in crack size, Δa. When the crack size is a, the TLM and the transmitted wave group travel $L_1 + L_2$ and $L_1 + L_3$ distances respectively, to reach the receivers. When the crack size increases by Δa, the extra distances travelled by the TLM and the transmitted Lamb mode are $2\Delta a$ and Δa respectively. In other words, the Turning Lamb Mode travels a greater distance than the transmitted Lamb mode while the crack propagates. Since the distance travelled by the Lamb mode is directly proportional to the change in arrival times (equations (2) and (4)), the difference in the arrival time of the TLM is more than that of the transmitted Lamb mode for a given crack growth, Δa. From this it can also be inferred that the TLM is more sensitive than the transmitted Lamb mode for gauging a given crack growth. A large difference in the arrival times of the TLMs than the transmitted Lamb modes can also be seen in Tables 2 and 3. This discussion surmises that the TLM is more sensitive to crack growth and is the right choice to predict crack growth.

5. Conclusions

A technique based on Turning Lamb Modes has been proposed to estimate the crack growth for the determination of G_{IC} in composite laminates. Numerical studies revealed that the TLM and transmitted Lamb mode based crack prediction techniques are viable solutions for the determination of G_{IC} with greater accuracy. An inference has also been made that the fundamental anti-symmetric TLM is more responsive to crack growth than the fundamental symmetric TLM.

References

Autar, K. Kaw, 1997, Mechanics of composite materials, CRC press.

Islam, A. S. and Craig, K. C., 1994, Damage detection in composite structures using piezoelectric materials, Smart Material and Structures, 3:318–28.

Karim, M. R., Mal, A. K. and Bar-Cohen Y., 1990, Inversion of leaky Lamb wave data by simplex algorithm. Journal of Acoustical Society of America, 88:482-491.

Lowe, M. J. S., 2003, Imperial College, London, UK., Disperse version 2.0.16b.

Raghavan, A. and Cesnik, C. E. S., 2007, Review of guided wave structural health monitoring. Shock Vibration Digest 39(2): 91-114.

Ramadas, C., Krishnan, Balasubramaniam, Makarand, Joshi and Krishnamurthy C. V., 2011, Composite Structures, 93: 1929-38.

Rao N S, 1997, Inverse problems in ultrasonic non-destructive characterization of composite materials using genetic algorithm, M. S Thesis, Mississippi State University, USA.

Rogers, W. P., 1995, Elastic property measurement using Rayleigh-Lamb waves, Journal of Research in Non-destructive Evaluation, 6: 185-208.

Rose, J. L., 1999, Ultrasonic Waves in Solid Media, Cambridge University Press, Cambridge.

Sridharan, Srinivasan ed., 2010, Delamination behavior of composites, Woodhead Publishing Ltd, CRC Press.

Vishnuvardhan, J., Krishnamurthy, C. V. and Krishnan, Balasubramaniam, 2007, Genetic algorithm based reconstruction of the elastic moduli of orthotropic plates using an ultrasonic guided wave single-transmitter-multiple-receiver SHM array, Smart materials and structures, 16:1639-1650.

Yang, C., Ye, Lin, Su, Z. and Bannister, M., 2006, Some aspects of numerical simulations for Lamb wave propagation in composite laminates, Composite Structures, 75: 267 – 275.

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